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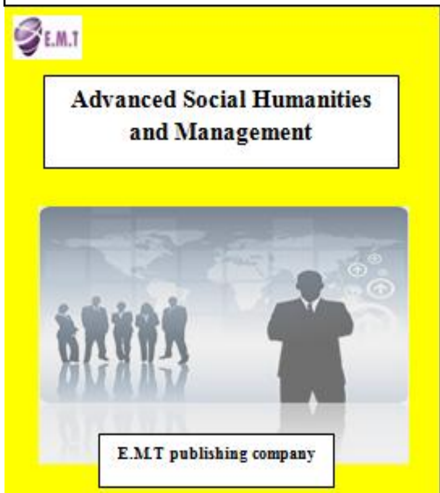
Solving a kind of fuzzy volterra integral equation using differential transforms method

H.Seiheii[1], S.Sohrabikia[2], K.Ghorbanpoor[3]
Department of Mathematics,Payamenoor University, Andimeshk, Iran
[1]E-mail address: hadis_seiheii@yahoo.com
[2]E-mail address:s.sohrabikia@gmail.com
[3]E-mail address: katira_ghorbanpoor@yahoo.com



Abstract:

In this paper we use parametric form of fuzzy number and convert a linear fuzzy volterra integral equation that kerenel is as $k(x-t) \geq 0.$ to two linear system of integral equation of the second kind in crisp case. We approximate the kernel of fuzzy integral equation with Taylor series and make fuzzy integral equation simpler by using some techniques that when we use differential transform method, we do not need difficult computation. Such that without this technique, solving integral equation by DTM method becomes hard. Through some examples, we have shown the application of these techniques and differential transform method. then we can use of the differential transform(DTM) method find the approximation solution of the system and hence obtain an approximation for fuzzy solution of the linear fuzzy volterra integral equations of the second kind.

Keywords: Differential transform method; fuzzy Volterra integral equation; Numerical method

1. Introduction

The concept of fuzzy numbers and arithmetic operations with this numbers were first introduced and investigated by Zadeh [9] and others.
S. Abbasbandy et al. [3] suggested a new Numerical method for solving linear Fredholm fuzzy integral equations of the second kind. The topics of fuzzy integral equations (FIE) which attracted growing interest for some time, in particular in relation to fuzzy control, have been developed in recent years. Goetschel and Vaxman [24] suggested a new approach, they represented the fuzzy number as a parameterized triple and then embedded the set of fuzzy numbers into a topological vector space. The establishment of the embedding banach space and its induced metric over its subset of fuzzy numbers led to immediate applications such as fuzzy least square [11,25,38,41,42], fuzzy linear system [6,7], fuzzy eigenvalues and eigenvectors [6,23]. Further applications such as solving fuzzy integral equations requested appropriated and applicable definitions of the fuzzy function and the fuzzy integral of fuzzy function. Congxin and Ming [39] represent the first applications of fuzzy integration. Numerical solution of linear Fredholm fuzzy integral equations of the second kind by Adomian method was introduced by Babolian et al. [5]. There are several research papers about obtaining the numerical integration of fuzzy-valued functions and solving fuzzy Volterra and Fredholm integral equations [18, 19, 20].
The differential transform method (DTM) is a numerical method for solving differential equations. The concept of the differential transform was first proposed by Zhou [28], Differential transform method for solving Volterra integral equation with separable kernels was introduced by
Zaid M. Odibat[45]. S. Salahshour et al. [44] suggested a new method for solving fuzzy Volterra integral equations.
This paper can be used to convey to students the idea that the DTM is a powerful tool for approximately solving fuzzy linear Volterra integral equation of the second kind that kerenel is as $k(x-t) \geq 0.$

2. Preliminaries

Here we recall the basic notations for symmetric fuzzy numbers and symmetric fuzzy linear systems.

Definition 1 [5, 31]. A fuzzy number is a map $u\colon \mathbb{R} \to I = [0,1]$ which satisfies

(i) u is upper semi-continuous.
(ii) $u(x) = 0$ Outside some interval $[c, d] \subset \mathbb{R}$.
(iii) There exist real numbers a, b such that $c \leq a \leq b \leq d$ where
1. $u(x)$ is monotonic increasing on $[c, a]$.
2. $u(x)$ is monotonic decreasing on $[b, d]$.
3. $u(x) = 1$, $a \leq x \leq b$.
The set of all such fuzzy numbers is represented by$\mathbb{E}$. an equivalent parametric definition of fuzzy numbers is given in [25, 32].

Definition 2. [4] An arbitrary fuzzy number in parametric form is represented by an ordered pair of functions $\left(\underline{u}(r), \overline{u}(r)\right)$, $0 \leq r \leq 1$, which satisfy the following requirements:
1. $\underline{u}(r)$ is a bounded left-continuous non-decreasing function over [0, 1].
2. $\overline{u}(r)$ is a bounded left-continuous non-increasing function over [0, 1].
3. $\underline{u}(r) \leq \overline{u}(r), 0 \leq r \leq 1$.

For arbitrary $u = \left(\underline{u}(r), \overline{u}(r)\right), v = \left(\underline{v}(r), \overline{v}(r)\right)$ and $k \in \mathbb{R}$ we define addition and multiplication by k as

$\underline{(u+v)}(r) = \left(\underline{u}(r), \underline{v}(r)\right),$
$\overline{(u+v)}(r) = \left(\overline{u}(r), \overline{v}(r)\right),$
$\underline{ku}(r) = k\underline{u}(r), \quad \overline{ku}(r) = k\overline{u}(r) \qquad if \quad k \geq 0,$
$\overline{ku}(r) = k\underline{u}(r), \quad \underline{ku}(r) = k\overline{u}(r) \qquad if \quad k < 0.$

Remark 1. [4] Let $u(r) = \left(\underline{u}(r), \overline{u}(r)\right)$, $0 \leq r \leq 1$ be a fuzzy number, we take

$u^c(r) = \dfrac{\underline{u}(r) + \overline{u}(r)}{2}, \qquad u^d(r) = \dfrac{\overline{u}(r) - \underline{u}(r)}{2}.$

It is clear that $u^d(r) \geq 0$ and $\underline{u}(r) = u^c(r) - u^d(r)$and $\overline{u}(r) = u^c(r) + u^d(r)$,
also a fuzzy number $u \in \mathbb{E}$ is said symmetric if $u^c(r)$ is independent of r for all
$0 \leq r \leq 1$.

Remark 2. [4]Let $u(r) = \left(\underline{u}(r), \overline{u}(r)\right), v(r) = \left(\underline{v}(r), \overline{v}(r)\right)$ and also k, s are arbitrary real numbers. If $w = ku + sv$ then
$w^c(r) = k\,u^c(r) + sv^c(r),$
$w^d(r) = |k|u^d(r) + |s|v^d(r).$

Definition 3. For arbitrary fuzzy numbers$u, v \in \mathbb{E}$, we use the distance [25]
$D(u,v) = \underbrace{sup}_{0 \leq r \leq 1} \max\left\{|\overline{u}(r) - \overline{v}(r)|, \left|\underline{u}(r) - \underline{v}(r)\right|\right\}$
And it is shown that $(\mathbb{E}, D)$ is a complete metric space [39].

Remark 3. [4]By referring to Remark 1 we have
$|\overline{u}(r) - \overline{v}(r)| \leq |u^c(r) - v^c(r)| + |u^d(r) - v^d(r)|$,
$|\underline{u}(r) - \underline{v}(r)| \leq |u^c(r) - v^c(r)| + |u^d(r) - v^d(r)|$
Hence for all $r \in [0, 1]$
$\max\{|\overline{u}(r) - \overline{v}(r)|, |\underline{u}(r) - \underline{v}(r)|\} \leq |u^c(r) - v^c(r)| + |u^d(r) - v^d(r)|$
And then
$D(u, v) \leq \underbrace{sup}_{0 \leq r \leq 1} \{|u^c(r) - v^c(r)| + |u^d(r) - v^d(r)|\}$.
Therefore if $|u^c(r) - v^c(r)|$ and $|u^d(r) - v^d(r)|$ tend to zero then $D(u, v)$ tend to zero.

Definition 4. ([21, 25]). Let $f: [a, b] \to \mathbb{E}$, for each partition $p = \{t_0, t_1, \ldots, t_n\}$ of $[a, b]$ and for arbitrary $\xi_i \in [t_{i-1}, t_i], 1 \leq i \leq n$ suppose

$$R_P = \sum_{i=1}^{n} f(\xi_i)(t_i - t_{i-1}),$$

$$\Delta := max\{|t_i - t_{i-1}|, i = 1, \ldots, n\}.$$

The definite integral of $f(t)$ over [a,b] is

$$\int_{\boldsymbol{a}}^{\boldsymbol{b}} f(t)dt = \lim_{\Delta \to 0} R_P$$

Provided that this limit exists in the metric D.If the fuzzy function $f(t)$ is continuous in the metric$D$, its definite integral exists [25], and also,

$$\overline{(\int_a^b f(t;r)dt)} = \int_a^b \overline{f}(t;r)dt,$$

$$\underline{\left(\int_a^b f(t;r)dt\right)} = \int_a^b \underline{f}(t;r)dt.$$

3. Fuzzy integral equation and basic idea of differential transform method

The Volterra integral equation of the second kind is an integral equation of the form:

$$u(x) = f(x) + \int_0^x k(x, t)u(t)dt, \tag{1.3}$$

In this study we survey the kind of fuzzy volterra integro-differential equation in the following form:

$$u(x) = f(x) + \int_0^x k(x - t)u(t)dt, \tag{2.3}$$

Where $\lambda \geq 0$, kerenel is as $k(x - t) \geq 0$, we investigate solution of fuzzy Volterra integral equations. Let $u(x)$ be a fuzzy-valued function to be solved for, $f(x)$ is given known function, and $k(x - t)$ a known real-valued integral kernel. If $f(t)$ is a crisp function then the solution of above equation is crisp as well. However, if $f(t)$ is a fuzzy function this equation may only possess fuzzy solution. Sufficient conditions for the existence equation of the second kind, where $f(t)$ is a fuzzy function, are given in [42]. For solving Eq. (1) in parametric form we take [4]

$$\underline{u}(x; r) = \lambda \int_0^x \underline{k(x - t)u(t; r)}\, dt, \tag{3.3}$$

$$\overline{u}(x; r) = \lambda \int_0^x \overline{k(x - t)u(t; r)}\, dt, \tag{4.3}$$

Hence

$$u^c(x; r) = \lambda \int_0^x k(x - t)u^c(t; r)\, dt,$$

$u^d(x;r) = \lambda \int_0^x |k(x-t)|\, u^d(t;r)dt, \qquad \|k(x-t)\| < 1,$

By referring to Remark 2 we have

$$u^c(x;r) = f^c(x,r) + \lambda \int_0^x k(x-t)u^c(t;r)dt, \tag{5.3}$$

$$u^d(x;r) = f^d(x;r) + \int_0^x |k(x-t)|u^d(t;r)dt, \tag{6.3}$$

It is clear that we must solve two crisp volterra integral equation of the second kind provided that each of Eqs. (4) And (5) have solution. We achieve Taylor series of kerenel and we put it in the equation (5) and (6). We segregate $u(x)$ into $u_0(x), u_1(x), \dots, u_n(x)$.

The differential transform of the derivative of afunction is defined as follows [42]

$$F(k) = \frac{1}{k!}\left[\frac{d^k f(x)}{dx^k}\right]_{x=x_0} \tag{7.3}$$

Where $f\,(x)$the original is function and $F(k)$ is the transformed function. The differential inverse transform of $F(k)$ is defined as

$$f(x) = \sum_{k=0}^{\infty} F(k)(x-t)^k \tag{8.3}$$

From Eqs. (7.3) and (8.3), we get

$$f(x) = \sum_{k=0}^{\infty} \frac{(x-t)^k}{k!} \frac{d^k f(x)}{dx^k}\bigg|_{x=x_0} \tag{9.3}$$

Which implies that the concept of differential transform is derived from Taylor series expansion, but the method does not evaluate the derivatives symbolically. However, relative derivatives are calculated by an iterative way which are

Described by the transformed equations of the original function. In real applications, the function $f\,(x)$ is expressed by a finite series and Eq. (2.2) can be written as

$$f(x) = \sum_{k=0}^{n} F(k)(x-t)^k, \tag{10.3}$$

Here n is decided by the convergence of natural frequency.

We achieve Taylor series of kernel.

Let $(x-t) = v\ \ and\ (x_0 - t_0) = v_0$, we have obtained:

$$k(v) = k(v_0) + k'(v_0)(v-v_0) + k''(v_0)\frac{(v-v_0)^2}{2!} + \cdots \tag{11.3}$$

We insert (2.5) in the equation (1.3):

$$u^c(x) = f^c(x) + \int_0^x (k(v_0) + k'(v_0)(v-v_0) + \cdots)u^c(t)dt. \tag{12.3}$$

Then,

We segregate $u^c(x)$ in to $u_0^c(x), u_1^c(x), \dots, u_n^c(x)$.

$$u_0^c(x) = f^c(x), \tag{13.3}$$

$$u_1^c(x) = \int_0^x k(v_0)u^c(x)\ dt, \tag{14.3}$$

$$u_2^c(x) = \int_0^x k'(v_0)(v-v_0)u^c(x)\ dt,$$

$$u_3^c(x) = \int_0^x k''(v_0)\frac{(v-v_0)^2}{2!}u^c(x)\ dt,$$

$$u_4^c(x) = \int_0^x k'''(v_0)\frac{(v-v_0)^3}{3!}u^c(x)\ dt,$$

$$\dots \tag{15.3}$$

By using differential equations (2.9), we obtain:

$$\frac{du_2^c(x)}{dx} = \frac{d}{dx}\int_0^x k'(v_0)(v-v_0)u^c(x)\ dt,$$

$$\frac{d^2u_3^c(x)}{dx^2} = \frac{d^2}{dx^2}\int_0^x k''(v_0)\frac{(v-v_0)^2}{2!}u^c(x)\ dt,$$
$$\frac{d^3u_4^c(x)}{dx^3} = \frac{d^3}{dx^3}\int_0^x k'''(v_0)\frac{(v-v_0)^3}{3!}u^c(x)\ dt,$$
$$\dots \tag{16.3}$$

Finally by using DTM equations (2.7), (2.8) and (2.10),
We will obtain $U_0^c(k), U_1^c(k), \dots, U_n^c(k)$. , and insert them in
$U^c(k) = \sum_{i=0}^n U_i^c(k)$. (17.3)
and similarly for $U^d(k)$:
$U^d(k) = \sum_{i=0}^n U_i^d(k)$. (18.3)

Theorem 1. [33, 45]Suppose that $U(k)$ is the differential transformations of the function$u(x)$,
$if\ \ f(x) = \int_{x_0}^x u(t)dt$
$then\ , F(k) = \dfrac{U(k-1)}{k}$

Theorem 2. $if\ f(x) = \frac{d^n u(x)}{dx^n}\ ,\ then\ F(k) =$
$(k+1)(k+2)\dots(k+n)U(k+n)$

Theorem 3.
$if\ \ f(x) = u(x) \pm v(x)\ \ ,then\ \ F(k) = U(k) \pm V(k)$

Theorem 4.
$$if\ \ f(x) = x^m\ ,\ then\ \ F(k) = \delta(k-1) = \begin{cases} 1 & if \quad k = m \\ 0 & if \quad k \neq m \end{cases}$$

Proof. The proof follows immediately from the definitions (7.3) and (8.3) and the operations of differential transformation given in[10] .

4. Numerical Examples

Many numerical techniques have been used successfully for such equations and in this section we discuss in detail a straightforward yet generally applicable technique: differential transform method.[4]

Example 1 [44]. Consider the following fuzzy volterra integral equation:
$u(x) = f(x) + \int_0^x (x-t)u(t)dt,$ (1.4)
Where $f(x;r) = [3+r, 8-2r]$. Based on equations (5.3) , (6.3) and using properties of DTM, we have for $0 \le r \le 1, k \ge 1$:
$f^c = \frac{11-r}{2}$, $f^d = \frac{5-3r}{2}$
$u^c(x) = \frac{11-r}{2} + \int_0^x (x-t)u^c(t)dt,\quad u^d(x) = \frac{5-3r}{2} + \int_0^x |(x-t)|u^d(t)dt,$
$|(x-t)| = (x-t)\ , because\ \ 0 \le t \le x$ and $k(x-t) \ge 0,$
using differential of $u^c(x)\ and\ u^d(x)$, we obtain

$\frac{du^c(x)}{dx}=\int_0^x u^c(t)dt,\quad \frac{du^d}{dx}=\int_0^x u^d(t)dt,$

Using properties of DTM, we have the following recurrence relation:

$(k+1)U^c(k+1)=\frac{U^c(k-1)}{k}\quad,\quad (k+1)U^d(k+1)=\frac{U^d(k-1)}{k}$

$U^c(0)=\frac{11-r}{2},\ \ U^c(1)=0\ ,U^c(2)=\frac{11-r}{4}\ ,U^c(3)=0\ ,U^c(4)=\frac{11-r}{48},\dots$

$U^d(0)=\frac{5-3r}{2}\ ,U^d(1)=0\ ,U^d(2)=\frac{5-3r}{4},U^d(3)=0,U^d(4)=\frac{5-3r}{48},\dots$

$\underline{U}(0;r)=3+r\ ,\underline{U}(1;r)=0,\underline{U}(2;r)=\frac{3+r}{2}\ ,\underline{U}(3;r)=0\ ,\underline{U}(4;r)=\frac{3+r}{24}$

$\underline{U}(5;r)=0\ ,\underline{U}(6;r)=\frac{3+r}{720}\ ,\underline{U}(7;r)=0,\underline{U}(8;r)=\frac{3+r}{40320},\dots$

$\overline{U}(0;r)=8-2r,\overline{U}(1,r)=0\ ,\overline{U}(2;r)=\frac{8-2r}{2}\ ,\overline{U}(3;r)=0\ ,\overline{U}(4;r)=\frac{8-2r}{24}$

$\overline{U}(5;r)=0\ ,\overline{U}(6;r)=\frac{8-2r}{720},\overline{U}(7;r)=0\ ,\overline{U}(8;r)=\frac{8-2r}{40320},\dots$

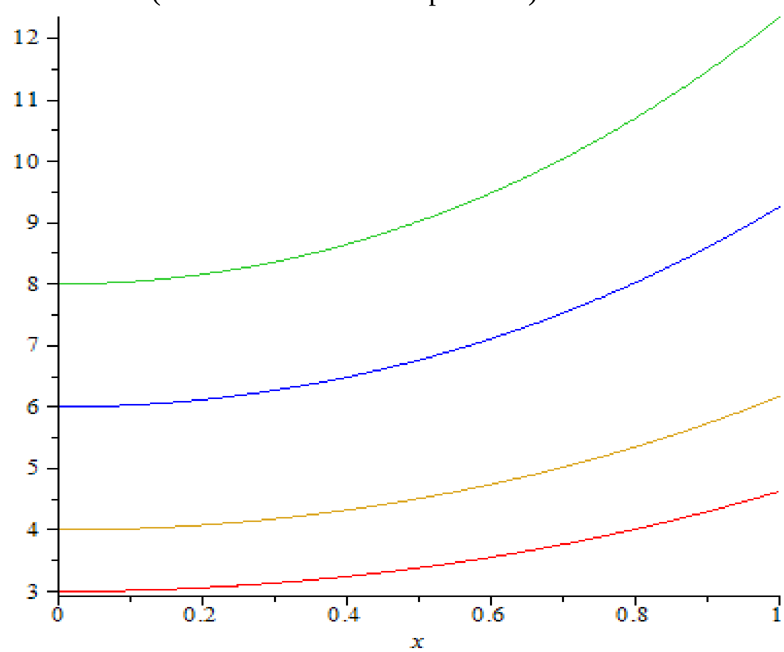
Based on the fact that

$\cosh(x)=1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\frac{x^8}{8!}+\cdots$

The r-cut representation of solution of (1.4) is

$u(x;r)=[3+r,8-2r]\odot\cosh(x),$

This is the exact solution of the fuzzy Volterra integral Eq. (1.4). Moreover, we plot the obtained solution (see Fig. 1) based on the 0-cuts and 1-cuts (obtained solution is trapezoidal).



x∈[0,1]

Fig. 1. Solution of Example1.

Example 2: Consider the following fuzzy volterra integral equation:

$$u(x)=[1,2-r]+\int_0^x \sin(x-t)\,u(t)dt, \qquad (2.4)$$

Where $f(x)=[1,2-r]$, Based on equations (5.3) , (6.3) and using properties of DTM, we have for $0\le r\le 1, k\ge 1$.

$f^c = \frac{3-r}{2}$, $\quad f^d = \frac{1-r}{2}$

$u^c(x) = \frac{3-r}{2} + \int_0^x \sin(x-t)\, u^c(t)dt,$

$u^d(x) = \frac{1-r}{2} + \int_0^x |\sin(x-t)| u^d(t)dt,$

$\sin(x-t) = \sum_{n=0}^{\infty} (-1)^n \frac{(x-t)^{2n+1}}{(2n+1)!} = (x-t) - \frac{(x-t)^3}{3!} + \frac{(x-t)^5}{5!} - \cdots$

$\sin(x-t) \cong (x-t) - \frac{(x-t)^3}{3!}$

$when\ k(x-t) \geq 0,$

$|\sin(x-t)| \cong \left|(x-t) - \frac{(x-t)^3}{3!}\right| = (x-t) - \frac{(x-t)^3}{3!},$

$u^c(x) = \frac{3-r}{2} + \int_0^x \left((x-t) - \frac{(x-t)^3}{3!}\right) u^c(t)dt$,

$u^d(x) = \frac{1-r}{2} + \int_0^x ((x-t) - \frac{(x-t)^3}{3!}) u^d(t)dt$,

$u_0^c(x) = \frac{3-r}{2}$, $\quad u_0^d(x) = \frac{1-r}{2}$,

$u_1^c(x) = \int_0^x (x-t) u^c(t)dt$, $\quad u_1^d(x) = \int_0^x (x-t) u^d(t)dt$,

$u_2^c(x) = -\int_0^x \frac{(x-t)^3}{3!} u^c(t)dt$, $\quad u_2^d(x) = -\int_0^x \frac{(x-t)^3}{3!} u^d(t)dt$,

Using differential of $u_1^c(x), u_2^c(x)$ and $u_1^d(x), u_2^d(x)$ we obtain:

$\frac{du_1^c(x)}{dx} = \int_0^x u^c(t)dt$, $\quad \frac{du_1^d(x)}{dx} = \int_0^x u^d(t)dt$,

$\frac{d^3 u_2^c(x)}{dx^3} = -\int_0^x u^c(t)dt$, $\quad \frac{d^3 u_2^d(x)}{dx^3} = -\int_0^x u^d(t)dt$,

Using properties of DTM, we have the following recurrence relation:

$U_0^c(k) \frac{3-r}{2} \delta(k),$ $\quad U_0^d(k) = \frac{1-r}{2} \delta(k),$

$(k+1) U_1^c(k+1) = \frac{U^c(k-1)}{k},$ $\quad (k+1) U_1^d(k+1) = \frac{U^d(k-1)}{k},$

$(k+1)(k+2)(k+3) U_2^c(k+3) = -\frac{U^c(k-1)}{k},$

$(k+1)(k+2)(k+3) U_2^d(k+3) = -\frac{U^d(k-1)}{k},$

$U^c(k) = \sum_{i=0}^{n} U_i^c(k),$ $\quad U^d(k) = \sum_{i=0}^{n} U_i^d(k),$

$U^c(0) = \frac{3-r}{2},$ $\quad U^d(0) = \frac{1-r}{2},$

$U^c(1) = 0,$ $\quad U^d(1) = 0,$

$U^c(2) = \frac{3-r}{4},$ $\quad U^d(2) = \frac{1-r}{4},$

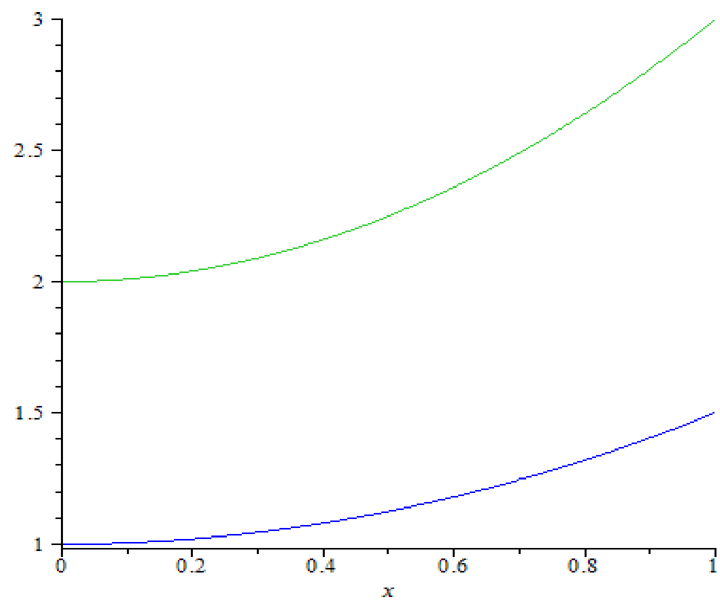
$U^c(3) = 0,$ $\quad U^d(3) = 0,$

…

$\overline{U}(0) = 2 - r,$ $\underline{U}(0) = 1,$
$\overline{U}(1) = 0,$ $\underline{U}(1) = 0,$
$\overline{U}(2) = \frac{2-r}{2},$ $\underline{U}(2) = \frac{1}{2},$
$\overline{U}(3) = 0,$ $\underline{U}(3) = 0,$
…

$u(x;r) \cong [1,2-r] \odot (1 + \frac{x^2}{2})$

This is the exact solution of the fuzzy Volterra integral Eq. (2.4). Moreover, we plot the obtained solution (see Fig. 2) based on the 0-cuts and 1-cuts,



x∈[0,1]
Fig. 2. Solution of Example 2.

5. Conclusions

In this paper, a differential approach and a numerical method successfully used for solving linear fuzzy volterra integral equation that kerenel is as $k(x-t) \geq 0$.
The numerical and (DTM) methods are faster than other methods and are less heavy computing needs. All calculations show that approximate solutions to solve fuzzy volterra integral equations of second kind, are more accurate and appropriate.

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