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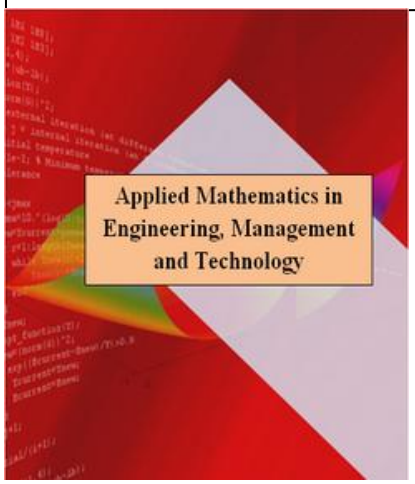
# Select the optimal portfolio using one-factor model

**Hamidreza Abedinzadeh ,** Department of Accounting, Yazd Branch, Islamic Azad University, Yazd, Iran
**Abolfazl Darehzereshki** , Department of Accounting, Young Researchers and Elites Club, Yazd Branch, Islamic Azad University, Yazd, Iran




**Abstract**

The capital markets are considered as main pillars of each country and their purpose is collecting small capitals and transferring them to producers. Investment shall be done such to provide the highest efficiency or lowest risk. One of the most complicated and challenging topics in financial literature is securities management and investment factors of market, means the factors or irregular conditions of investor, it seems that with a proper planning and knowledge of the relationship between risk and return, many uncertainties of investment will be resolved and the problem of choosing investment by a decision maker can be solved.
In this study we have attempted to choose the optimal portfolio among companies selected in Tehran stock exchange, using the one-factor model. In order to select the optimal portfolio in Tehran stock exchange, the excess return rate to beta, ranking stocks and the determining cut rate in two modes;( a) based on bank interest rate, 12% and b) based on interest rate on bonds, 16%) are determined after calculating research variables. Then with selecting the adequate and suitable sample, the stocks in the composition of the optimal portfolio are selected and at last the ratio of investment in each stock is determined in both modes.

**Key words**: Investment, One-Factor Model, Beta Coefficient, Portfolio Selection, Optimal Portfolio

## Introduction

One of the biggest challenges that investors face it in the financial markets is making decision about how to invest. The investment is based on two important issues. First, it is the amount of investment in a particular stock and the determined stock storage time. Investors face the uncertainty (risk) in investment process in financial markets and try to reduce overall risk of portfolio with selecting different stocks. One of the ways to optimize portfolio is using one-factor model. This model was formed on the basis of a new definition of risk. Risk factors that constitute a change in the price of market stocks will be ranked in categories of political risk, commercial risk, interest rate risk, inflation risk, liquidity risk, financial risk and currency risk (Raie & Telengi, 2001). The growing commercial and trading activities, as well as uncertainty and instability in financial markets cause to promote and develop the studies of risk and with increasing the demand for participation in financial and investment markets, led to develop the methods to measuring the risk. One of the risk assessment criteria that were common during the last twenty years is using the sensitive Beta coefficient. Using this coefficient to facilitate the portfolio management process is based on modern portfolio theory (MPT), since the calculation of this process due to too complex and time consuming till before the popularity and easier use, the Beta coefficient was utilized. (Raie & Saeedi, 2001). In 1961 William F. Sharpe[1], based on the initial image of Markowitz model[2], provided one-factor model[3] and the concept of stuck portfolio. In his opinion, the stuck risk is separated to two systematic and non-systematic, in which its systematic part is controllable and the other one is uncontrollable, also its amount will be equal to market portfolio. Sharpe defined the Beta criteria for risk and using it explained the portfolio risk as a linear combination of each stock. One-factor mode is a statistical mode to express the process of creating efficiency. The basic concept in one-factor model is that all securities are affected by public volatility of market.

One-factor model assumptions are as follow: (Eslami Bidgoli & Heibati, 1995)

[1] Sharpe William F
[2] Markowitz
[3] Single index Model

1- Evaluation criteria for investment projects, the expected return and standard deviation over a period of time.
2- Investors are unsaturated and in choosing between two schemes (portfolio) pay attention to efficiently plan.
3- Investors are risky and in choosing between two schemes, are interested to lower risk.
4- The assets are divided and analyzed unlimitedly.
5- There is a risk-free rate of lending / borrowing.
6- There are no transaction costs and taxes.

The aim of this study is selecting the optimal portfolio in Tehran stock exchange using a one-factor model.

## 2- Research Generalities

### 2.1. Illustration of Research Issue

Choosing portfolio has always been one of the most important issues in investment management and also is continued. Since Markowitz solved this problem with his Mean-variance[4] model for the first time, numerous surveys has been conducted in this area. Efforts to achieve more effective strategies for stick selection and optimal portfolio construction composed by different models will help to increase the efficiency and reduce investors risk and definitely become practical the investment process and stock assessment (Mousavi Zadeh, 2006)

Risk and return are two main issues in investment discussion, and in this case there are two important points of view that according to which, firstly, investors bear the risk only when they get excess return and second, the risk reduce with a variety (Laftthose[5], 1994).

Securities have variable yields that their level of volatility is measured by the variance and standard deviation. The decision making to invest in integrated level is one of the most complex and the most challenging issues in securities management and investment analysis, so that it has the highest returns in the same risk or the lowest risk in terms of the same efficiency. As a result, if possible, taking into account the market factors, means uncontrollable conditions and factors, investor can choose the appropriate and optimal model to invest under the uncertainty conditions of investment, a decision maker can solve the problem of investment portfolio selection.

The trend of this study is such that, firstly we consider the model has been presented by Sharpe for optimizing the investment portfolio as the research basis and then obtain our objectives and provide our propositions with calculating the excess return ration to companies stocks Beta ranking according, determination of cut-off rate and companies sticks selection to include in optimal portfolio and finally to determine the investment percent of per share and choosing the optimal portfolio.

### 2.2. Research Type

This project was done with aim of using the results of findings to select the stock optimal portfolio for investors; its results can be guide to investor's action, the portfolio managers in making their investment and also authorities and actors of capital market. Therefor this study is an applied research objectively.

### 2.3. Methodology and Research Scope

This research was conducted to select the investment optimal portfolio by decision maker. The model used to construct the optimal portfolio is Sharpe's one-factor model. While this study is based on the applied purpose and due to the descriptive data collection, it is used with aim of facilitating the decision-making process, in other words the general approach of this research is analytical-descriptive method.

**Subject Scope**: including the topics and concepts of optimal portfolio construction.

**Research period**: this period lasted from 2002 to 2007.

[4] Mean - Variance
[5] Laftthose

**Static community**: it consists of all accepted companies listed in Tehran stock exchange; these ones are in the accepted companies list of Tehran stock exchange at the end of 2007. (317 companies)
**Research sample**: consists of 33 companies which their stocks are traded more than the average amount of year working days during the period of research. This sample was randomly selected. The name of firms that have been selected as sample mentioned in table 1.
Table 1: The companies studied in this research

| company Code | Company Name | company Code | Company Name | company Code | Company Name |
|---|---|---|---|---|---|
| A01 | Offset | A12 | Saipa Diesel | A23 | Behshahr Industry |
| A02 | Iran Tiers | A13 | Melli Bank Investment | A24 | Iran Marine Industrial |
| A03 | Iran Khodro Diesel | A14 | Caspian cement | A25 | Food products and sugar of Piranshahr |
| A04 | Iranmerinos | A15 | Sepahan Cement | A26 | Shirin Sugar Khorasan |
| A05 | Behnoosh . Iran | A16 | Sina Darou | A27 | Alborz cable |
| A06 | Pars drug | A17 | Drug Company | A28 | Iran Carton |
| A07 | Paksan | A28 | Zahravi Pharmaceutical companies | A29 | Esfahan Tile |
| A08 | Providing Foundry Sand | A219 | Glass & bottle | A30 | Floors |
| A09 | Iran Tractor Co. | A20 | Shahid Bahonar Copper Industries | A31 | Iran Combine |
| A10 | Iran Chinaware | A21 | Azarab Industry | A32 | Butane Industrial Group |
| A11 | Iran tractor Casting | A22 | AMA Industry | A33 | Motogen |

### 2.4. Data Collection Method of Research

One of the main methods of data collection in this study is library research.
So that the theoretical discussion needed for research was collected the related sources including books, articles, theses as well as available sources in databases and libraries of universities and higher education institutions. Information needed to conduct research was used through securities exchanges websites such as WWW.IRBOURSE.COM and www.TSETMC.com and other sites as well as information software such as Dena and Sahra.

### 2.5. Necessity and Importance of Research

The purpose of an appropriate investment is obtaining the best return tighter with the lowest portfolio risk by optimization process of investment combination. At the time of decision making, investors divide to two groups, experience (aware) and without experience (non-specialist) in selecting the appropriate series of securities and subsequently the portfolio selection make in two ways. First, securities series selection carry out by investors with sufficient knowledge in this field and second, providing parameters needed by investor to agent or an specialist and portfolio selection by investor. In which the necessity of this study can be called the procedural and selecting the securities and optimal portfolio to both groups, especially the latter.

### 2.6. Research Result Application

The main use of this study results is utilizing one-factor model in stock investment management of companies listed on Tehran stock exchange. So, small investors, institutional investors, investment found and other investors in securities exchange are able to manage their investment use the procedures and results of this research in order to choose the optimal portfolio.

### 2.7. Research Question

The main question in this study is that due to the uncertainty prevailing in Tehran stock exchange market, what will be the optimal portfolio using one-factor model?

## 3- Description Statistics

According to the information related to the last price of stocks during the period of research, the daily stock returns were calculated. Evaluation of descriptive statistics indicates that the average return per day is, 0.073%. However, the stock changes in Tehran stock exchange are asymmetric and its negative changes will be more than positive ones.
In results of Kolmogorov- Smirnov test indicate a non-normal distribution of data, since the convention $sig_s$ obtained 0.000 and is less than 5%. Therefore the efficiency distribution of companies in the study was not normal in 95% of certainty and does not follow this distribution.

## 4- Optimal Portfolio Construction Algorithm Using One-Factor Model

The most common method to estimate parameters in one-factor model is using historical return rates. For example, having an annual return rate if stock 1 as well as the return of market portfolio in the last year would be time series regression (Raiee & Talengi, 2003):

(1) $$R_{it} = \alpha_i + \beta_i R_{mt} + e_{it}$$

(2)
(3)
(4)
$$\beta_i = \frac{\sum_{t=1}^{n}(R_{it} - \bar{R}_i)(R_{mt} - \bar{R}_m)}{\sum_{t=1}^{n}(R_{mt} - \bar{R}_m)^2}$$

$$\alpha_i = \bar{R}_i - \beta_i R_m$$

$$e_{it} = R_{it} - \bar{R}_{it}$$

$\alpha_i$: is independent factor of return in a function of market factor and random variable

$R_i$: is return of per sheet of stocks

$R_m$: is the rate of market factor return (index) that is a random variable

$\beta_i$: Changes expected in $R_i$ for any change in $R_m$ are measured

Therefore to conduct an optimal portfolio using one-factor model, the following steps should be considered:
1- Calculation of related variables (return, beta, variance residual. etc) for ranking securities.
2- Ranking stocks of the studied companies to determine their suitability for inclusion in optimal portfolio (based on excess return in beta).
3- Determining the cut rate of companies stock under the study and their calculation.
4- Determining the percent of investment and optimal portfolio creation.

### 4.1. Optimal Portfolio Construction

If we consider the standard one-factor model as a describing model for synchronize changes among securities, in this mode, the appropriateness of each stock depends on the ratio of stock excess return to its Beta. The excess return is the difference between the expected return on stocks and free-risk internet rate. The ratio of excess return to Beta can measure the excess return of stock sheet per unit change in non-diversification risk. In other words, an index used to rank the stocks, is called "excess return to Beta", which is expressed in mathematical language as follows: (Raie & Talengy, 2002):

(5) $$\frac{(\overline{R}_i - R_f)}{\beta_i}$$

Where:

$R_f$ is return of being risk.

Rules to select stock in optimal portfolio combination are as follows: (Raie & Talengy, 2002):

1- Ratio of excess return to Beta for all stock under consideration is calculated and sorted from the highest to the lowest.

2- Optimal portfolio includes investment in all securities in which its $(\overline{R}_i - R_f)/\beta_i$ ratio will be higher than the cut-off point, $C^*$, means:

(6) $$\frac{\overline{R}_i - R_f}{\beta_i} > C_i \Rightarrow$$

**4.2. Ranking Securities**

Suitability of per stock for inclusion in optimal portfolio depends on the excess return of stock to be Beta. In other words, as the securities rank based on the "ratio of excess return to Beta" (in order from highest to lowest), the obtained result indicate the suitability of per stock in portfolio. It is worth mentioning that due to not equaling the free-risk interest rate 12% and 16%, in this research both of them has been used to from an optimal portfolio, so the results show that the ranking of securities about companies under study is the same in both a): (12% interest rate of bank) and b): (16% interest rate of bonds) and have no different with other.

Table 2: ranking results of companies stocks to determine their suitability

| Stock code | Expected return | $\beta_i$ | $\delta_{ei}^2$ | ratio of excess return to beta (based On) | |
|---|---|---|---|---|---|
| | | | | 12% | 16% |
| A32 | -0.00531 | -0.11524 | 0.00037 | 1.08738 | 1.43448 |
| A31 | -0.00066 | -0.25034 | 0.00012 | 0.48198 | 0.64177 |
| A28 | -0.00641 | -0.36175 | * 9.7 E -05 | 0.34944 | 0.46002 |
| A29 | -0.00104 | -0.79301 | 3.9 E -06 | 0.15263 | 0.20307 |
| A18 | -0.00183 | -0.90014 | 8.3 E -06 | 0.13535 | 0.17978 |
| A23 | -0.02023 | -1.14505 | 1.2 E -05 | 0.12247 | .1574 |
| A24 | -0.00452 | -1.28421 | 4.3 E -05 | 0.09696 | 0.12811 |
| A12 | -0.00182 | -2.03789 | 1.2 E -05 | 0.05978 | 0.07941 |
| A08 | -0.00628 | -2.52007 | 7.9 E -05 | 0.05011 | 0.06598 |
| A14 | -0.00566 | -5.25138 | 4.8 E -05 | 0.02393 | 0.03155 |
| A30 | 0.022049 | -5.80955 | 0.00313 | 0.01686 | 0.02375 |
| A01 | -0.95848 | -85.4384 | 0.17399 | 0.01262 | 0.01309 |
| A02 | 0.047901 | 13.7566 | 0.00015 | -0.00524 | -0.00815 |

| Stock code | Expected return | $\beta_i$ | $\delta_{ei}^2$ | ratio of excess return to beta (based On) | |
|---|---|---|---|---|---|
| | | | | 12% | 16% |
| A13 | 0.001193 | 4.10674 | 0.00028 | -0.02893 | -0.03867 |
| A16 | -0.00144 | 3.1379 | 1.6 E - 05 | -0.0387 | -0.05145 |
| A21 | -0.0012 | 2.92896 | 1.8 E - 05 | -0.04138 | -0.05504 |
| A04 | 0.014102 | 1.78295 | 0.00012 | -0.05939 | -0.08183 |
| A11 | -0.00604 | 1.98259 | 4.6 E - 05 | -0.06357 | -0.08375 |
| A17 | 0.001565 | 1.76273 | 2.5 E - 06 | -0.06719 | -0.08988 |
| A20 | -0.00724 | 1.00035 | 0.00035 | -0.12719 | -0.16718 |
| A06 | -0.00075 | .9142 | 2.4 E - 06 | -0.13208 | -0.17584 |
| A05 | 0.004687 | 0.73929 | 5.2 E - 06 | -0.15598 | -0.21008 |
| A19 | -0.00196 | 0.65882 | 1.9 E - 05 | -0.18512 | -0.24583 |
| A22 | -0.00084 | .5491 | 1.3 E - 05 | -0.22007 | -0.29291 |
| A03 | -0.00034 | 0.40869 | 6.4 E - 06 | -0.29445 | -0.39233 |
| A09 | 0.00096 | .3053 | 4.7 E - 05 | -0.38991 | -0.52093 |
| A15 | -0.00231 | 0.30458 | 1.6 E - 05 | -0.40157 | -0.5329 |
| A26 | 0.00043794 | 0.28549 | 6.2 E - 08 | -0.41879 | -0.5589 |
| A27 | -0.0029 | 0.12964 | 0.0001 | -0.94801 | -1.25656 |
| A10 | -0.00237 | 0.10053 | 8.5 E - 06 | -1.21721 | -1.61509 |
| A07 | -0.00117 | .0989 | 1 E - 05 | -1.22521 | -1.62967 |
| A33 | 0.00067 | 0.09205 | 4. 1 E - 06 | -1.29639 | -1.73095 |
| A25 | -0.00146 | 0.04904 | 9.9 E - 06 | -2.47661 | -3.29222 |

### 4.3. Determination of Cut-Off Rate

In calculating the amount of $C^*$, all features of the securities available in optimal portfolio will be used. If $C_i$ is a candidate elected as $C^*$, it's worth is determined when the stock 1 applied in optimal portfolio.
A favorable $C_i$ (ie, $C^*$) must be specified such that all stocks with excess return ratio to a Beta higher than $C_i$ include in $C_i$ calculation and the stocks with excess return ratio to Beta lower than $C_i$, do not notably used.

### 4.4. Calculation of Cut-Off Rate

Suppose that all stocks under consideration are ranked based on the ratio of excess return to beta. In this case for portfolio with stock i. $C_i$ appointed as follow (Raie & Talengy, 2002):

$$C_i = \frac{\sigma_m^2 \sum_{j=1}^{i} \frac{(\bar{R}_j - R_f)\beta_i}{\sigma_{ej}^2}}{1 + \sigma_m^2 \sum_{j=1}^{i} \frac{(\beta_j^2)}{\sigma_{ej}^2}} \quad (7)$$

Where:

$\sigma_m^2$ : is variance of market index, and

$\sigma_{ej}^2$ : is variance of stock changes that is independent of market index changes and called non-systematic risk.

After determining the suitability of companies stock to include it in optimal portfolio using one-factor model application, it is necessary to determine which company's stock can apply in optimal portfolio combination.

### 4.4.1. Determining the Cut-Off Rate Based on Bank Interest Rate (12%)

The comparison between amounts of column $C_i^*$ in table 3 with numerical values of calculations about "excess return ration" in table 2 indicates that only stocks of seven companies mentioned in the top of table 3, can select in combination of portfolio and stocks of other companies are eliminated.

Table 3: determining the stock cut-off rate of companies under study (based on 12%)

| Stock Rank | Stock Code | $C_i$ (Based on 12% interest rate of Bank) | Stock Rank | Stock Code | $C_i$ (Based on 12% interest rate of Bank) |
|---|---|---|---|---|---|
| 1 | A32 | 9.09887 E - 05 | 18 | A11 | 0.008161733 |
| 2 | A31 | 0.000688995 | 19 | A17 | -0.008354485 |
| 3 | A28 | 0.001766534 | 20 | A20 | -0.008414195 |
| 4 | A29 | 0.042216169 | 21 | A06 | -0.015679489 |
| 5 | A18 | 0.055361993 | 22 | A05 | -0.018084255 |
| 6 | A23 | 0.064491247 | 23 | A19 | -0.018710902 |
| 7 | A24 | 0.065965294 | 24 | A22 | -0.019491677 |
| 8 | A12 | 0.064191022 | 25 | A03 | -0.020659848 |
| 9 | A08 | 0.063294298 | 26 | A09 | -0.020779099 |
| 10 | A14 | 0.050896351 | 27 | A15 | -0.021135311 |
| 11 | A30 | 0.050698001 | 28 | A26 | -0.091244729 |
| 12 | A01 | 0.049854589 | 29 | A27 | -0.091263236 |
| 13 | A02 | 0.028088861 | 30 | A10 | -0.091442102 |
| 14 | A13 | 0.027027512 | 31 | A07 | - |

| | | | | | |
|---|---|---|---|---|---|
| | | | | | 0.091589189 |
| 15 | A16 | 0.016328006 | 32 | A33 | -0.091919549 |
| 16 | A21 | 0.010016674 | 33 | A25 | -0.091997128 |
| 17 | A04 | 0.009574919 | | | |

### 4.4.2. Determining the Cut-Off Rate Based on Interest Rate of Bonds (16%)

Table 4: determining the stock cut-off rate of companies under study (based on 16%)

| Stock Rank | Stock Code | $C_i$ (Based on 16% Interest rate of bonds | Stock Rank | Stock Code | $C_i$ (Based on 16% Interest rate of bonds |
|---|---|---|---|---|---|
| 1 | A32 | 0.000120033 | 18 | A11 | 0.010285067 |
| 2 | A31 | 0.000916286 | 19 | A17 | -0.011670514 |
| 3 | A28 | 0.002334763 | 20 | A20 | -0.011748649 |
| 4 | A29 | 0.056155930 | 21 | A06 | -0.021388521 |
| 5 | A18 | 0.073606712 | 22 | A05 | -0.024622827 |
| 6 | A23 | 0.085006330 | 23 | A19 | -0.025452722 |
| 7 | A24 | 0.086963045 | 24 | A22 | -0.026489821 |
| 8 | A12 | 0.084796057 | 25 | A03 | -0.028044075 |
| 9 | A08 | 0.083597959 | 26 | A09 | -0.028203254 |
| 10 | A14 | 0.067204418 | 27 | A15 | -0.028675373 |
| 11 | A30 | 0.066951154 | 28 | A26 | -0.122157484 |
| 12 | A01 | 0.065758086 | 29 | A27 | -0.122181988 |
| 13 | A02 | 0.036560929 | 30 | A10 | -0.122419148 |
| 14 | A13 | 0.035160577 | 31 | A07 | -0.122614688 |
| 15 | A16 | 0.021062085 | 32 | A33 | -0.123055697 |
| 16 | A21 | 0.012739408 | 33 | A25 | -0.123158797 |
| | A04 | 0.012137544 | | | |

Results obtained from comparison of both cut-off rates, 12% and 16%, indicate that the number and type of stocks related to companies including in optimal portfolio are the same.

### 4.5. Construction of Optimal Portfolio

After determining all of the securities that must be included in optimal portfolio combination, this fact remains that: how much should be the ratio of investment in each stock sheet? The percent of investment per stock is equal to: (Raie & Talengy, 2002):

(8)

And $Z_i$ is equal to:

(9) $$Z_i = \frac{\beta_i}{\sigma_{ei}^2}\left(\frac{\bar{R}_i - R_f}{\sum_{i=1}^{n} Z_i \beta_i} - C^*\right)$$

According to equation (8), the total proportion of investment in securities is equal to one. In other words, the above equation provides the budget imitation of investor.

Table 5 shows amount of $Z_i$ for both 12% and 16% as well as the percentage of investment in bonds available in optimal portfolio.

Table 5: amount of $Z_i$ and percentage ratio per stock in optimal portfolio combination

| $\%X = \frac{Z_i}{\sum_{i=1}^{n} Z_i}$ | | $Z_i$ | | Stock code |
|---|---|---|---|---|
| 16% | A 12% | 16% | A 12% | |
| 0.9559308 | 0.9585788 | -425.44761 | -322.48811 | A32 |
| 2.6902481 | 2.6686999 | -1197.3248 | -897.81249 | A31 |
| 3.1264695 | 3.1429295 | -1391.4699 | -1057.3543 | A28 |
| 52.631122 | 51.971144 | -23,424.065 | -17484.297 | A29 |
| 22.722232 | 22.468755 | -10,112.781 | -7559.0098 | A18 |
| 15.12094 | 16.046218 | -6729.7422 | -5398.3194 | A23 |
| 2.7530578 | 2.7436756 | -1225.2789 | -923.03603 | A24 |
| 100 | 100 | -44506.109 | -33,642.317 | Sum |

Table 6 shows the investment percentage per stock including in optimal portfolio.

Table 6: percentage of investment in stocks of companies included in optimal portfolio

| Percent of investment | | Stock Name | Stock code |
|---|---|---|---|
| 16% | A 12% | | |
| 0.96 | 0.96 | Iran National Lead and zinc | A32 |
| 2.69 | 2.67 | Butane Industrial Group | A31 |
| 3.13 | 3.14 | Esfahan Tile | A28 |
| 52.63 | 51.97 | Floors | A29 |
| 22.72 | 22:47 | Glass & bottle | A18 |
| 15:12 | 16:05 | Iran Marine | A23 |

| Percent of investment | | Stock Name | Stock code |
|---|---|---|---|
| 16% | A 12% | | |
| | | Industrial | |
| 2.75 | 2.74 | Piranshahr Food products and sugar | A24 |

## 6. Conclusion

Using one-factor model for the construction of optimal portfolio, in addition to its modeling, has been easy; its variables are specific and understandable and have a high degree of interaction with users.
The main purpose of this research has been creating an optimal portfolio using one-factor model in Tehran stock exchange. In this regard, after the collection of data needed to achieve this objective by applying one-factor model, the optimal portfolio in Tehran stock exchange was established and created. The results of performing one-factor model to form the optimal portfolio in Tehran sticks exchange in both modes a): (based on 12% interest rate of bank) and b): (based on 16% interest rate of bonds) indicate that among stocks of companies under study, only stocks of seven companies includes in optimal portfolio combination.
Since in this research, the one-factor model to create the optimal portfolio has been used and despite the market index, the other different factors are affective on stock return, so it offers to carry out another research to create and select the optimal portfolio using multi-factors model in Tehran stocks stock exchange.

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