Abstract
In this paper, the Economic Dispatch (ED) problem has been optimized by Imperialist Competitive Algorithm (ICA). ICA is a population based stochastic optimization technique, originally developed by Eberhart and Kennedy, inspired by simulation of a social psychological metaphor instead of the survival of the fittest individual. In ICA, the system (imperialists) is initialized with a population of random solutions (colonies) and searches for optima using cognitive and social factors by updating generations. The proposed methodology is validated for a test system consisting of 13 thermal units whose incremental fuel cost function takes into account the valve-point loading effects.

Keywrod: Economic dispatch, Imperialist competitive algorithm, Power system, Optimization.

1. INTRODUCTION

The objective of the Economic Dispatch (ED) problem of electric power generation, whose characteristics are complex and highly nonlinear, is to schedule the committed generating unit outputs so as to meet the required load demand at minimum operating cost while satisfying all unit and system equality and inequality constraints [1]. Improvements in scheduling the unit outputs can lead to significant cost savings. In traditional ED problems, the cost function of each generator is approximately represented by a simple quadratic function and is solved using mathematical programming based on several optimization techniques, such as dynamic programming, linear programming, homogenous linear programming, and nonlinear programming techniques [2]. However, none of these methods may be able to provide an optimal solution, for they usually get stuck at a local optimum.

The published works of ED problem have been classified in four groups; which are: evolutionary algorithm, Particle Intelligence Technique (PIT), fuzzy methods and heuristic techniques. Evolutionary Algorithm (GA), Immune Algorithm (IA), Harmony Search (HS) algorithm and Differential Evolutionary (DE) algorithm are the most powerful and best-known evolutionary techniques. GA has been used to solve ED problem in [3-5]. Ref.[3] introduces a new generic equation for the economic dispatch model of power systems, which include conventional generators as well as wind powered generators considering the variable nature of wind velocity and power demand. Ref.[4] aims to concentrate on the online economic dispatch of MECs for which it provides a probabilistic ED optimization model. The presented model is treated via a robust optimization technique, i.e., multiagent genetic algorithm (MAGA), whose outstanding feature is to find well the global optima of the ED problem. Basu in [5] presents nondominated sorting genetic algorithm-II for solving fuel constrained economic emission dispatch problem of thermal generating units. This is a multi-objective optimization problem which includes the standard load constraints as well as the fuel constraints. IA is other evolutionary technique which has been employed to optimize the ED problem. Authors in [6] have presented an algorithm inspired on the T-Cell model of the immune system which is used to solve economic dispatch problems. Ref.[7] presents a novel heuristic algorithm to solve dynamic economic dispatch problem of generating units by employing a hybrid immune-genetic algorithm.

In [8-10], ED problem has been solved by HS algorithm. A simple yet efficient HS method with a new pitch adjustment rule (NPAHS) is proposed in [8] for dynamic economic dispatch of electrical power systems, a large-scale non-linear real time optimization problem imposed by a number of complex constraints. Ref.[9] presents a new multi-objective HS algorithm for environmental/economic dispatch problem. Ref.[10] presents the hybrid HS algorithm with swarm intelligence to solve the dynamic economic load dispatch problem.
Finally in [14-16], DE algorithm has been introduced as solution technique. Ref.[14] presents an improved DE to solve economic dispatch problem of thermal generating units with non-smooth/non-convex cost functions due to valve-point loading taking into account transmission losses and nonlinear generator constraints such as prohibited operating zones. In [15], a DE algorithm combined with truncated Lévy flight random walks and a population diversity measure to improve the crossover and mutation operations is designed to help avoiding premature convergence effectively. Ref.[16] presents a multi-objective DE algorithm for environmental/economic ED problem.

Three PITs have been proposed to solve ED problem; these techniques are: Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO) and Artificial Bee Colony (ABC) algorithms. Authors in [11-13], PSO algorithm has been suggested to find optimal solution of ED problem. Ref.[11] presents a novel parameter automation strategy for PSO algorithm for solving non-convex emission constrained economic dispatch problems. Ref.[12] presents modified PSO to solve economic dispatch problems with non-smooth/non-convex cost functions. In [13], quantum-inspired PSO (QPSO) is proposed, which has stronger search ability and quicker convergence speed, not only because of the introduction of quantum computing theory, but also due to two special implementations: self-adaptive probability selection and chaotic sequences mutation.

ACO algorithm has been used by authors of Ref.[17-19] to solve ED problem. Ref.[17] has developed a multi-objective ACO method for solving the multi-objective EED problems of thermal generators in power systems. Ref.[18] presents a novel ACO approach for this problem. The main characteristics of the ACO algorithm are positive feedback, distributed computation and the use of a constructive greedy heuristic. Ref.[19] presents a novel and efficient optimization approach based on the ACO for solving the ED problem with non-smooth cost functions.

ABC is other issue of PITs which in [20-22] has been proposed to solve ED algorithm Liao et al in [20] present a novel adaptive Algorithm and compare its efficiency with other existing algorithms for long-term dispatch of cascaded hydropower systems. In [21], Gbest guided ABC algorithm is applied to optimize the emission and overall cost of operation of wind–thermal power system. In [22], incremental ABC and incremental ABC with local search have been used for the solution of the economic dispatch problem with valve point effect.

Fuzzy techniques in [23-25] has been suggested to optimize ED problem. An improved DE method is proposed in [23] where the selection operation is modified to reduce the complexity of multi-attribute decision making with the help of a fuzzy framework. Ref.[24] presents an interactive fuzzy satisfying method for solving an ED problem assuming that the decision maker (DM) has imprecise or fuzzy goals for each of the objective functions. Ref.[25] explains a new improvement in the DT technique by adding fuzzy logic (FL) to the unit limits and load (FLDT).

Finally in [26-28], three heuristic methods have been proposed to find optimal solution for ED problem. In [26], an environmental economic dispatch problem with storage, network, and inter-temporal constraints is considered. Ref.[27] presents a distributed algorithm based on auction techniques and consensus protocols to solve the non-convex economic dispatch problem. Authors in [28] have formulated the economic dispatch problem as a two-stage stochastic convex program for operational decision making under uncertainty.

An economic dispatch problem with 13 unit test system using non smooth fuel cost function [29] is employed in this paper for demonstrate the performance of the proposed hybrid method. The results obtained with the ICA approach were analyzed and compared with those obtained in recent literature. The rest of the paper is organized as follows: SectionII describes the ED problem, while Section III explains the ICA concepts. Section IV presents the simulation results of the 13 thermal units whose incremental fuel cost function takes into account the valve-point loading effects. Lastly, Section V outlines our conclusion.

II. DESCRIPTION OF ECONOMIC DISPATCH PROBLEM

The objective of the economic dispatch problem is to minimize the total fuel cost at thermal power plants subjected to the operating constraints of a power system. Therefore, it can be formulated mathematically with an objective function and two constraints. The equality and inequality constraints are represented by equations (1) and (2) given by:

\[ \sum_{i=1}^{n} P_i - P_L - P_0 = 0 \]  
\[ P_{i_{\text{min}}} \leq P_i \leq P_{i_{\text{max}}} \]
In the power balance criterion, an equality constraint must be satisfied, as shown in equation (1). The generated power should be the same as the total load demand plus total line losses. The generating power of each generator should lie between maximum and minimum limits represented by equation (2), where \( P_i \) is the power of generator \( i \) (in MW); \( n \) is the number of generators in the system; \( P_D \) is the system’s total demand (in MW); \( P_L \) represents the total line losses (MW) and \( P_i^{\max} \) and \( P_i^{\min} \) are, respectively, the output of the minimum and maximum operation of the generating unit \( i \) (MW). The total fuel cost function is formulated as follows:

\[
\min f = \sum_{i=1}^{n} F_i(P_i)
\]

where, \( F_i \) is the total fuel cost for the generator unity \( i \) (in $/h), which is defined by equation:

\[
F_i(P_i) = a_i P_i^2 + b_i P_i + c_i
\]

where, \( a_i, b_i \) and \( c_i \) are cost coefficients of generator \( i \). The generator costs are usually approximated using quadratic functions. However, it is more practical to consider the valve-point loading for fossil-fuel-based plants. In this context, a cost function is obtained based on the ripple curve for more accurate modeling. This curve contains higher order nonlinearity and discontinuity due to the valve point effect. One way of representing this effect is to model the generator cost curve by piecewise quadratic cost functions. A second approach is using a rectified sinusoidal function to represent the valve-point loading in the cost function \[1\]. In this case, equation (4) can be modified, as:

\[
\tilde{F}_i(P) = F(P) + e_i \sin \left( f_i \left(P_i^{\min} - P_i\right)\right)
\]

or

\[
\tilde{F}_i(P) = a_i P_i^2 + b_i P_i + c_i + e_i \sin \left( f_i \left(P_i^{\min} - P_i\right)\right)
\]

where, \( e_i \) and \( f_i \) are coefficients of the valve points effect. Ignoring valve point effects, some inaccuracy would be introduced into the resulting dispatch. Hence, the total fuel cost that must be minimized, according to equation (3), is modified to:

\[
\min f = \sum_{i=1}^{n} \tilde{F}_i(P_i)
\]

where, \( \tilde{F}_i \) is the cost function of generator \( i \) (in $/h) defined by equation (6). In the case study presented here, we disregarded the transmission losses, \( P_L \); thus, \( P_L = 0 \).

### III. OPTIMIZATION METHODS

#### A. THE PROPOSED ALGORITHM

Figure 1 shows the flowchart of the proposed algorithm like other evolutionary ones, the proposed algorithm starts with an initial population (countries in the world). Some of the best countries in the population are selected to be the imperialists and the rest form the colonies of these imperialists. All the colonies of initial population are divided among the mentioned imperialists based on their power. The power of an empire which is the counterpart of the fitness value in GA, is inversely proportional to its cost. After dividing all colonies among imperialists, these colonies start moving toward their relevant imperialist country. The way by which they move is described in section C. The total power of an empire depends on both the power of the imperialist country and the power of its colonies. We will model this fact by defining the total power of an empire by the power of imperialist country plus a percentage of mean power of its colonies. Then the imperialistic competition begins among all the empires. Any empire that is not able to succeed in this competition and can’t increase its power (or at least prevent decreasing its power) will be eliminated from the competition. The imperialistic competition will gradually result in an increase in the power of powerful empires and a decrease in the power of weaker ones. Weak empires will lose their power and ultimately they will collapse. The movement of colonies toward their relevant imperialists along with competition among empires and also the collapse mechanism will hopefully cause all the countries to converge to a state in which there exist just one empire in the world and all the other countries are colonies of that empire. In this ideal new world colonies, have the same position and power as the imperialist.
B. Generating Initial Empires

The goal of optimization is to find an optimal solution in terms of the variables of the problem. We form an array of variable values to be optimized. In GA terminology, this array is called “chromosome”, but here the term “country” is used for this array. In an $N_{\text{var}}$-dimensional optimization problem, a country is a $1 \times N_{\text{var}}$ array. This array is defined by,

$$
country = \begin{bmatrix} P_1 & P_2 & P_3 \ldots & P_{N_{\text{var}}} \end{bmatrix}
$$

The variable values in the country are represented as floating point numbers. The cost of a country is found by evaluating the cost function $f$ at the variables $(P_1, P_2, P_3, \ldots, P_{N_{\text{var}}})$. Then,

$$
cost = f(\text{country}) = f(P_1, P_2, P_3, \ldots, P_{N_{\text{var}}})
$$

To start the optimization algorithm we generate the initial population of size $N_{\text{pop}}$. We select $N_{\text{imp}}$ of the most powerful countries to form the empires. The remaining $N_{\text{col}}$ of the population will be the colonies each of which belongs to an empire. Then we have two types of countries; imperialis and colony.

To form the initial empires, we divide the colonies among imperialists based on their power. That is the initial number of colonies of an empire should be directly proportionate to its power. To divide the colonies among imperialist proportionally, we define the normalized cost of an imperialist by,

$$
C_i = c_i - \max \{c_i\}
$$
Fig. 1. Flowchart of the proposed algorithm

where, $c_n$ is the cost of $n$th imperialist and $C_n$ is its normalized cost. Having the normalized cost of all imperialists, the normalized power of each imperialist is defined by,

$$p_n = \frac{C_n}{\sum_{i=1}^{N_{imperialist}} C_i}$$  \hspace{1cm} (11)$$

From another point of view, the normalized power of an imperialist is the portion of coloniens that should be possessed by that imperialist. Then the initial number of colonies of an empire will be,

$$N.C_n = round \{p_n.N_{col}\}$$  \hspace{1cm} (12)$$
where $N.C_n$ is the initial number of colonies of $n$th empire and $N_{col}$ is the number of all colonies. To divide the colonies, for each imperialist we randomly choose $N.C_n$ of the colonies and give them to it. These colonies along with the imperialist will form $n$th empire.

C. Moving the Colonies of an Empire toward the Imperialist

We have modeled this fact by moving all the colonies toward the imperialist. This movement is shown in figure 2 in which the colony moves toward the imperialist by $x$ units. The new position of colony is shown in a darker color. The direction of the movement is the vector from colony to imperialist. In this figure $x$ is a random variable with uniform (or any proper) distribution. Then for $x$ we have,

$$x \sim U (0, \beta \times d)$$  \hspace{1cm} (13)

where $\beta$ is a number greater than 1 and $d$ is the distance between colony and imperialist. A $\beta > 1$ causes the colonies to get closer to the imperialist state from both sides.

![Fig. 2. Moving colonies toward their relevant imperialist](image)

To search different points around the imperialist we add a random amount of deviation to the direction of movement. Figure 3 shows the new direction. In this figure $\theta$ is a random number with uniform (or any proper) distribution. Then

$$\theta \sim U (-\gamma, \gamma)$$  \hspace{1cm} (14)

where $\gamma$ is a parameter that adjusts the deviation from the original direction. Nevertheless the values of $\beta$ and $\gamma$ are arbitrary, in most of our implementation a value of about 2 for $\beta$ and about $\pi/4$ (Rad) for $\gamma$ have resulted in good convergence of countries to the global minimum.

![Fig. 3. Moving colonies toward their relevant imperialist in a randomly deviated direction.](image)

D. Exchanging Positions of the Imperialist and a Colony

While moving toward the imperialist, a colony may reach to a position with lower cost than that of imperialist. In such a case, the imperialist moves to the position of that colony and vise versa. Then algorithm will continue by the imperialist in a new position and then colonies start moving toward this position.

E. Total Power of an Empire

Total power of an empire is mainly affected by the power of imperialist country. But the power of the colonies of an empire has an effect, albeit negligible, on the total power of that empire. We have modeled this fact by defining the total cost by,
\[ T.C_n = cost(\text{imperialist}_n) + \xi \text{mean}(cost(\text{colonies of empire}_n)) \]  
(14)

where, \(T.C_n\) is the total cost of the \(n\)th empire and \(\xi\) is appositive number which is considered to be less than 1. A little value for \(\xi\) causes the total power of the empire to be determined by just the imperialist and increasing it will increase the role of the colonies in determining the total power of an empire. We have used the value of 0.1 for \(\xi\) in most of our implementation.

**F. Imperialistic Competition**

As mentioned in section II, all empires try to take possession of colonies of other empires and control them. This imperialistic competition gradually brings about a decrease in the power of weaker empires and an increase in the power of more powerful ones. We model this competition by just picking some (usually one) of the weakest colonies of the weakest empires and making a competition among all empires to possess these (this) colonies. Figure 4 shows a big picture of the modeled imperialistic competition. Based on their total power, in this competition, each of empires will have a likelihood of taking possession of the mentioned colonies. In other words these colonies will not be possessed by the most powerful empires, but these empires will be more likely to possess them.

To start the competition, first, we find the possession probability of each empire based on its total power. The normalized total cost is simply obtained by

\[ N.T.C_n = T.C_n - \max \{T.C_i\} \]  
(15)

The more powerful an empire is, the more likely it will possess the weakest colony of the weakest empire. where, \(T.C_n\) and \(N.T.C_n\) are respectively total cost and normalized total cost of \(n\) empire. Having the normalized total cost, the possession probability of each empire is given by

\[ P_{P_n} = \frac{N.T.C_n}{\sum_{i=1}^{N.T.C_i}} \]  
(16)

To divide the mentioned colonies among empires based on the possession probability of them, we form the vector \(P\),

\[ P = [P_{P_1}, P_{P_2}, P_{P_3}, \ldots, P_{P_{\text{imp}}} ] \]  
(17)

Then we create a vector with the same size as \(P\) whose elements are uniformly distributed random numbers.

\[ R = [r_1, r_2, r_3, \ldots, r_{N_{\text{imp}}} ] \]  
(18)

\[ r_1, r_2, r_3, \ldots, r_{N_{\text{imp}}} \sim U \]  
(19)

Then we form vector \(D\) by simply subtracting \(R\) from \(P\).

\[ D = P - R = [D_1, D_2, D_3, \ldots, D_{N_{\text{imp}}} ] \]  
(19)
Referring to vector $D$ we will hand the mentioned colonies to an empire whose relevant index in $D$ is maximum.

G. Eliminating the Powerless Empires

Powerless empires will collapse in the imperialistic competition and their colonies will be divided among other empires. In modeling collapse mechanism different criteria can be defined for considering an empire powerless. In most of our implementation, we assume an empire collapsed and eliminate it when it loses all of its colonies.

H. Convergence

After a while all the empires except the most powerful one will collapse and all the colonies will be under the control of this unique empire. In this ideal new world all the colonies will have the same positions and same costs and they will be controlled by an imperialist with the same position and cost as themselves. In this ideal world, there is no difference not only among colonies but also between colonies and imperialist. In such a condition we put an end to the imperialistic competition and stop the algorithm.

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IV. CASE STUDY OF EDP WITH 13 THERMAL UNITS

This case study consisted of 13 thermal units of generation with the effects of valve-point loading, as given in Table I. The data shown in Table I are also available in [29-30]. In this case, the load demand expected to be determined was $P_D = 1800$ MW.

<table>
<thead>
<tr>
<th>Thermal unit</th>
<th>$p_{\text{min}}^i$</th>
<th>$p_{\text{max}}^i$</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>e</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>680</td>
<td>0.00028</td>
<td>8.1</td>
<td>550</td>
<td>300</td>
<td>0.035</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>360</td>
<td>0.00056</td>
<td>8.1</td>
<td>306</td>
<td>200</td>
<td>0.042</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>360</td>
<td>0.00056</td>
<td>8.1</td>
<td>307</td>
<td>150</td>
<td>0.042</td>
</tr>
<tr>
<td>4</td>
<td>60</td>
<td>180</td>
<td>0.00324</td>
<td>7.74</td>
<td>240</td>
<td>150</td>
<td>0.063</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
<td>180</td>
<td>0.00324</td>
<td>7.74</td>
<td>240</td>
<td>150</td>
<td>0.063</td>
</tr>
</tbody>
</table>
The simulation results obtained are given in Table II, which shows that the chaotic ICA succeeded in finding the best solution for the tested methods. However, the IF outperformed the other tested methods in terms of solution time. The best results obtained for solution vector $P_i$, $i=1,..,13$ with chaotic ICA with minimum cost of $17960.5358 \$/h$ is given in Table III.

Table II. Convergence results (50 runs) of a case study of 13 generating

<table>
<thead>
<tr>
<th>Optimization Technique</th>
<th>Case Study with 13 Thermal Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evolutionary programming [10]</td>
<td>17994.07</td>
</tr>
<tr>
<td>Particle swarm optimization [1]</td>
<td>18030.72</td>
</tr>
<tr>
<td>Hybrid evolutionary programming with SQP [1]</td>
<td>17991.03</td>
</tr>
<tr>
<td>Hybrid particle swarm with SQP [1]</td>
<td>17969.93</td>
</tr>
<tr>
<td>Hybrid Chaotic Particle Swarm Optimizer</td>
<td>17963.9571</td>
</tr>
<tr>
<td>Best result of this paper using ICA approach</td>
<td>17960.5358</td>
</tr>
</tbody>
</table>

Table III. Best results (4 runs) obtained using ICA approach

<table>
<thead>
<tr>
<th>Power</th>
<th>Generation (MW)</th>
<th>Power</th>
<th>Generation (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>628.3173124938128</td>
<td>$P_8$</td>
<td>60.000000000000000000</td>
</tr>
<tr>
<td>$P_2$</td>
<td>149.5996451155268</td>
<td>$P_8$</td>
<td>109.8500876611908</td>
</tr>
<tr>
<td>$P_3$</td>
<td>222.8040779498990</td>
<td>$P_{10}$</td>
<td>40.000000000000000000</td>
</tr>
<tr>
<td>$P_4$</td>
<td>109.8415858364943</td>
<td>$P_{11}$</td>
<td>40.000000000000000000</td>
</tr>
<tr>
<td>$P_5$</td>
<td>109.8580678527901</td>
<td>$P_{12}$</td>
<td>55.000000000000000000</td>
</tr>
<tr>
<td>$P_6$</td>
<td>109.8662358231647</td>
<td>$P_{13}$</td>
<td>55.000000000000000000</td>
</tr>
<tr>
<td>$P_7$</td>
<td>109.8629872671215</td>
<td>$\sum_{i=1}^{13} P_i$</td>
<td>1800.0000000000000000</td>
</tr>
</tbody>
</table>

Table IV compares the results obtained in this paper with those of other studies reported in the literature. Note that in studied case, the result reported here using chaotic ICA is comparatively lower than recent studies presented in the literature.

Table IV. Comparison of case study results for fuel costs presented in the lectures

<table>
<thead>
<tr>
<th>Optimization Method</th>
<th>Mean Time (s)</th>
<th>Minimum Cost ($/h$)</th>
<th>Mean Cost ($/h$)</th>
<th>Maximum Cost ($/h$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IF</td>
<td>1.4</td>
<td>18812.3852</td>
<td>18962.0139</td>
<td>19111.6426</td>
</tr>
<tr>
<td>PSO</td>
<td>2.6</td>
<td>18874.7634</td>
<td>19159.3967</td>
<td>19640.4168</td>
</tr>
<tr>
<td>Chaotic PSO</td>
<td>3.3</td>
<td>18161.1013</td>
<td>18809.8275</td>
<td>19640.7556</td>
</tr>
<tr>
<td>PSO-IF</td>
<td>14.8</td>
<td>18605.1257</td>
<td>18854.1601</td>
<td>19111.6426</td>
</tr>
<tr>
<td>Chaotic PSO-IF</td>
<td>15.3</td>
<td>17963.9571</td>
<td>18725.2356</td>
<td>19037.2663</td>
</tr>
<tr>
<td>ICA</td>
<td>-----</td>
<td>17960.5358</td>
<td>17963.3487</td>
<td>17967.9724</td>
</tr>
</tbody>
</table>

Conclusion
The contribution of this paper is the using of ICA to solve an ED problem. This method was applied to optimize for fuel cost of 13 thermal units. Results were analyzed and compared with other heuristic optimization algorithm. The results show that the proposed method has better influence than other heuristic optimization to solve an EDP. Another profit of the proposed method was the high accuracy and speed of that in reaching the optimal value.

Reference


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